Differential geometry of the space of Ising models  

BENJAMIN MACHTA, RICKY CHACHRA, Department of Physics, Cornell University, MARK TRANSTRUM, Department of Bioinformatics and Computational Biology at the MD Anderson Cancer Center, JAMES SETHNA, Department of Physics, Cornell University — We use information geometry to understand the emergence of simple effective theories, using an Ising model perturbed with terms coupling non-nearest-neighbor spins as an example. The Fisher information is a natural metric of distinguishability for a parameterized space of probability distributions, applicable to models in statistical physics. Near critical points both the metric components (four-point susceptibilities) and the scalar curvature diverge with corresponding critical exponents. However, connections to Renormalization Group (RG) ideas have remained elusive. Here, rather than looking at RG flows of parameters, we consider the reparameterization-invariant flow of the manifold itself. To do this we numerically calculate the metric in the original parameters, taking care to use only information available after coarse-graining. We show that under coarse-graining the metric contracts very anisotropically, leading to a “sloppy” spectrum with the metric’s Eigenvalues spanning many orders of magnitude. Our results give a qualitative explanation for the success of simple models: most directions in parameter space become fundamentally indistinguishable after coarse-graining.