Cycloid trajectory for a spin in a rotating magnetic field
SANGCHUL OH, XUEDONG HU, Department of Physics, University at Buffalo, The State University of New York — A cycloid is a curve traced by a point on the rim of a circle rolling on a straight (or in general, a base) line. In classical mechanics, it is known as the solution of two famous problems: the brachistochrone (least-time) curve and tautochrone (equal-time) curve. Here we show that a cycloid is the quantum trajectory on the Bloch sphere when a spin is dragged along by a rotating magnetic field. Here an imaginary circle, whose radius is determined by how fast the magnetic field is rotating, rolls on the base line of the rotating magnetic field on the Bloch sphere. If the magnetic field rotates slower, the radius of the rolling circle shrinks (to a point at the adiabatic limit, when the trajectory traces a circle that spans a solid angle proportional to the Berry phase). We find that like classical cycloid curves, the curtate cycloid on a Bloch sphere is generated for initial states within a circle on the Bloch sphere surface, and a prolate cycloid results from initial states outside of this circle. If the initial state is given by the center of the circle, the quantum trajectory is a line of a constant latitude on the Bloch sphere, parallel to the curve of the rotating magnetic field.

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