Probability Current in Hydrogen with Spin-Orbit Interaction
WILLIAM HODGE, Davidson College, SAM MIGIRDITCH, None, WILLIAM KERR, Wake Forest University — The spin-orbit interaction is a coupling between a particle’s spin and its motion. The Hamiltonian for a spin-1/2 particle which includes this coupling is

$$\mathcal{H} = \frac{\mathbf{p}^2}{2m} + V(x) + \frac{\nabla V(x) \times \mathbf{p}}{2m^2c^2} \cdot \mathbf{S}. \hspace{1cm} (1)$$

To describe the flow of probability in this system, we derive the continuity equation, which takes the usual form. In this case, however, we find the probability current density $\mathbf{j}(x, t)$ to be the sum of two terms. The first term is the one obtained by most quantum mechanics textbooks during their derivation of the continuity equation. The second term,

$$j_s(x, t) = \frac{1}{2m^2c^2} \sum_{\sigma, \sigma' = \uparrow, \downarrow} \left[ \psi^\ast(x, \sigma, t) \langle \sigma | \mathbf{S} | \sigma' \rangle \psi(x, \sigma', t) \times \nabla V(x) \right], \hspace{1cm} (2)$$

arises due to the inclusion of the spin-orbit term in the Hamiltonian and is small compared to the first. Using a perturbative treatment, we calculate $\mathbf{j}(x, t)$ for hydrogenlike atoms; for states with $\ell = 0$, we find that $\mathbf{j}(x, t) = j_s(x, t)$.