Quantum mechanics over sets DAVID ELLERMAN, Retired — In models of QM over finite fields (e.g., Schumacher’s “modal quantum theory” MQT), one finite field stands out, \( \mathbb{Z}_2 \), since \( \mathbb{Z}_2 \) vectors represent sets. QM (finite-dimensional) mathematics can be transported to sets resulting in quantum mechanics over sets or QM/sets. This gives a full probability calculus (unlike MQT with only zero-one modalities) that leads to a fulsome theory of QM/sets including “logical” models of the double-slit experiment, Bell’s Theorem, QIT, and QC. In QC over \( \mathbb{Z}_2 \) (where gates are non-singular matrices as in MQT), a simple quantum algorithm (one gate plus one function evaluation) solves the Parity SAT problem (finding the parity of the sum of all values of an \( n \)-ary Boolean function). Classically, the Parity SAT problem requires \( 2^n \) function evaluations in contrast to the one function evaluation required in the quantum algorithm. This is quantum speedup but with all the calculations over \( \mathbb{Z}_2 \) just like classical computing. This shows definitively that the source of quantum speedup is not in the greater power of computing over the complex numbers, and confirms the idea that the source is in superposition.