Theory of equilibria of elastic braids with applications to DNA supercoiling

GERT VAN DER HEIJDEN, EUGENE STAROSTIN, University College London — Motivated by supercoiling of DNA and other filamentous structures, we formulate a new theory for equilibria of 2-braids, i.e., structures formed by two elastic rods winding around each other in continuous contact and subject to a local interstrand interaction. Unlike in previous work no assumption is made on the shape of the contact curve. Rather, this shape is solved for. The theory is developed in terms of a moving frame of directors attached to one of the strands with one of the directors pointing to the position of the other strand. The constant-distance constraint is automatically satisfied by the introduction of what we call braid strains. The price we pay is that the potential energy involves arclength derivatives of these strains, thus giving rise to a second-order variational problem. The Euler-Lagrange equations for this problem give balance equations for the overall braid force and moment referred to the moving frame as well as differential equations that can be interpreted as effective constitutive relations encoding the effect that the second strand has on the first as the braid deforms under the action of end loads. Both open braid and closed braid solutions (links and knots) are computed and current applications to DNA supercoiling are discussed.

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