Compressed modes for variational problems in mathematical physics and compactly supported multiresolution basis for the Laplace operator

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