Breakdown of the scale invariance in a near-Tonks-Girardeau gas: some exact results and beyond ZHEDONG ZHANG, SUNY Stony Brook, GREGORY ASTRAKHARCHIK, Universitat Politècnica de Catalunya, STEVEN CHOI, University of Massachusetts Boston, HÉLÈNE PERRIN, Université Paris 13, THOMAS BERGEMAN, SUNY Stony Brook, MAXIM OLSHANII, University of Massachusetts Boston — In this presentation, we consider elementary monopole excitations of harmonically trapped one-dimensional Bose gas in the vicinity of a Tonks-Girardeau limit. Using the Girardeau Fermi-Bose mapping we obtain the first dominant correction to the excitation frequency, beyond the scale-invariance-protected value of $2\omega$. In limit of a large number of atoms, our result coincides with the upper bound predicted by Menotti and Stringari [Phys. Rev. A 66, 043610 (2002)]. We find further that, surprisingly, the frequency of the collective excitations, obtained using the perturbation theory [Phys. Rev. Lett. 81, 4541 (1998)], is found to be substantially below the Menotti-Stringari bound. In the latter case, the value of the frequency correction is $9/4$ times higher than in the former. Finally, an ab initio numerical calculation of the collective excitation frequency returns to the value predicted for the elementary excitation. We conjecture that the sharp boundary of the TG cloud, characterized by an infinite density gradient, renders the perturbation theory for the collective excitation frequencies unapplicable. We also discuss an extension of our results to the case of spin-polarized $p$-wave-interacting fermions in a cold waveguide.