Exact solutions of fractional Chern insulators: interacting particles in the Hofstadter model at finite size

THOMAS SCAFFIDI, STEVEN SIMON, University of Oxford — We show that all the bands of the Hofstadter model on the torus have an exactly flat dispersion and Berry curvature when a special system size is chosen. This result holds for any hopping and Chern number. Our analysis therefore provides a simple rule for choosing a particularly advantageous system size when designing a Hofstadter system whose size is controllable, like a qubit lattice or an optical cavity array. The density operators projected onto the flat bands obey exactly the Girvin-MacDonald-Platzman algebra, like for Landau levels in the continuum in the case of $C = 1$, or obey its straightforward generalization in the case of $C > 1$. This allows a mapping between density-density interaction Hamiltonians for particles in the Hofstadter model and in a continuum Landau level. By using the well-known pseudopotential construction in the latter case, we obtain fractional Chern insulator phases, the lattice counterpart of fractional quantum Hall phases, that are exact zero-energy ground states of the Hofstadter model with certain interactions. Finally, the addition of a harmonic trapping potential is shown to lead to an appealingly symmetric description in which a new Hofstadter model appears in momentum space.