Hyperuniformity Length in Experimental Foam and Simulated Point Patterns

ANTHONY CHIECO, ADAM ROTH, University of Pennsylvania, REMI DREYFUS, CNRS-Solvay-UPenn, SALVATORE TORQUATO, Princeton University, DOUGLAS DURIAN, University of Pennsylvania — Systems without long-wavelength number density fluctuations are called hyperuniform (HU). The degree to which a point pattern is HU may be tested in terms of the variance in the number of points inside randomly placed boxes of side length L. If HU then the variance is due solely to fluctuations near the boundary rather than throughout the entire volume of the box. To make this concrete we introduce a hyperuniformity length h, equal to the width of the boundary where number fluctuations occur. Thus h helps characterize the disorder. We show how to deduce h from the number variance, and we do so for Poisson and Einstein patterns plus those made by the vertices and bubble centroids in 2d foams. A Poisson pattern is one where points are totally random. These are not HU and h equals L/2. We coin “Einstein patterns” to be where points in a lattice are independently displaced from their site by a normally distributed amount. These are HU and h equals the RMS displacement from the lattice sites. Bubble centroids and vertices are both HU. For these, h is less than L/2 and increases slower than linear in L. The centroids are more HU than the vertices, in that h that increases more slowly.