Shear Jamming in Granular Media
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We numerically study two-dimensional packings of bidisperse disks created using isotropic compression and simple shearing protocols. To create jammed packings by compression, we start with $N$ particles with random initial positions and grow their diameters by successive small packing fraction increments each followed by relaxation of particle overlaps using energy minimization until the system cannot be compressed further without particle overlaps in the relaxed state. Jammed packings created via isotropic compression exist over a range of packing fractions $\phi$. Because of the spread of jammed packing fractions, during compression the system may reach a packing fraction above the minimum value before jamming. If an unjammed packing is then sheared by a strain $\gamma$, it can jam. Using a combination of compression and shearing, we can define jamming protocols as trajectories in the $(\phi, \gamma)$ plane that yield jammed packings. In this plane, we can reach a particular point $(\phi_n, \gamma_n)$ in many ways. We will focus on two of these: (1) shearing to $\gamma_n$ at $\phi = 0$ followed by compression to $\phi_n$ at $\gamma = \gamma_n$ and (2) compression to $\phi_n$ at $\gamma = 0$ followed by shearing to $\gamma_n$ at $\phi = \phi_n$. For protocol 1, we find that the probability of obtaining a jammed packing at $\phi$ and $\gamma$, $P(\phi, \gamma) = Q(\phi)$, is independent of $\gamma$. For protocol 2, we use a simple theory to deduce $P(\phi, \gamma)$ from $Q(\phi)$. Furthermore, we find that frictionless jammed packings form one-dimensional families in the $(\phi, \gamma)$ plane. The one-dimensional families are projections from the $2N$-dimensional configuration space onto the $(\phi, \gamma)$ plane. If the system reaches a given packing fraction by isotropic compression without jamming, the system will eventually “hit” one of the families and jam during shear. In packings composed of frictionless particles, the range of accessible jammed packing fractions shrinks with increasing $N$. However, in packings composed of frictional disks, we have shown that the families are no longer one-dimensional, the range of jammed packing fractions is broad even for large $N$ and depends on the number of missing contacts $m$. Therefore, the theoretical predictions used above for packings of frictionless disks must be modified to explain packings formed under shear in frictional systems. We predicted and measured the probability of forming a jammed packing with friction coefficient $\mu$ and $m$ missing contacts, $P_m(\mu)$. Here, we extend these studies to include the dependence of the jamming probability on packing fraction and shear strain, $P_m(\mu, \phi, \gamma)$, and relate this function to shear-jammed packings in frictional systems.

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