Learning probabilities from random observables in high dimensions: the maximum entropy distribution and others

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\textsuperscript{1}I am a member of one of the reciprocal societies, The Physical Society of Japan (JPS), and put the ID of JPS above.

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Date submitted: 03 Nov 2015  Electronic form version 1.4