Semiparametric energy-based models of systems exhibiting criticality  

JAN HUMPLIK, GASPER TKACIK, Institute of Science and Technology Austria — Over the last decade, several empirical studies have found evidence that many biological and natural systems exhibit critical fluctuations analogous to those observed during second-order phase transitions in equilibrium systems. In many cases, these fluctuations were shown to be equivalent to a thermodynamic version of Zipf’s law— if the system is sufficiently large, then a log-log plot of the probability of a state vs. its rank yields a straight line with slope −1. Because the origin of critical fluctuations cannot be traced to a unique mechanism, it is important that data-driven phenomenological models of natural systems are flexible enough so as to easily capture any kind of criticality. Here we study a class of models with exactly this property. This class consists of energy-based models in which the exponential Boltzmann factor is replaced by an arbitrary nonlinear function. We demonstrate the usefulness of our method by modeling the spiking activity of a population of retinal neurons, and the distribution of light intensities in small patches of natural images. In light of recent work on models with hidden variables, the proposed method can separate interactions induced by an unknown fluctuating environment from interactions intrinsic to the system.