Natural generalization of Slater determinants to more than one dimension

DENIS SUNKO, Department of Physics, Faculty of Science, University of Zagreb

— The calculation of realistic \( N \)-body wave functions for identical fermions is still an open problem in physics, chemistry, and materials science, even for \( N \) as small as two. Here a fundamental algebraic structure of many-body Hilbert space is described, enabling theoretically well-founded systematic investigation of wave-function space. The structure allows an arbitrary many-fermion wave function to be written in terms of a finite number of antisymmetric functions called shapes, which cannot be constructed by combining one-dimensional wave functions. Shapes naturally generalize the single-Slater-determinant form for the ground state to more than one dimension. Their number is exactly \( N!^{d-1} \) in \( d \) dimensions. A general algorithm is given to list them all in terms of standard Slater determinants. Conversely, excitations which can be induced from the one-dimensional case are bosonised into a system of distinguishable bosons, called Euler bosons, much like the electromagnetic field is quantized in terms of photons distinguishable by their wave numbers. Their wave functions are given explicitly in terms of elementary symmetric functions, reflecting the fact that the fermion sign problem is trivial in one dimension. The shapes are all possible vacua for the Euler bosons.

Denis Sunko
Department of Physics, Faculty of Science, University of Zagreb

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