Towards all electrical control of topological Josephson junctions and Majorana zero modes via spin-orbit interactions XIN LIU, School of Physics and Wuhan National High Magnetic Field Center, Huazhong University of Science and Technology, Wuhan, Hubei 430074, China, XIAOPENG LI, Condensed Matter Theory Center and Joint Quantum Institute, Department of Physics, University of Maryland, College Park, MD 20742-4111, USA, XIONG-JUN LIU, International Center for Quantum Materials and School of Physics, Peking University, Beijing 100871, China, DONG-LING DENG, Condensed Matter Theory Center and Joint Quantum Institute, Department of Physics, University of Maryland, College Park, MD 20742-4111, USA — We study the current-phase relation of topological Josephson junctions with spin-orbit interactions, and show that the coupling between Majorana zero modes (MZMs) can be controlled via gate tunable spin-orbit couplings (SOCs). The spin-triplet pairings in the presence of MZMs at the two ends of a one-dimensional topological superconductor, are shown to have a $\pi$ phase difference, from which a Josephson $\pi$-junction can be created. This $\pi$ phase is unambiguously manifested to be a spin-dependent superconducting phase, dubbed spin-phase. We demonstrate that SOC can induce such spin-phase in spin-triplet superconducting condensates which can tune the MZM coupling energy and allow a finite topological Josephson current without a magnetic flux in superconducting circuits. We further establish the linkage between this Josephson current and the fermion parity in a topological Josephson junction and propose an all-electronically controlled superconductor-semiconductor hybrid circuit to detect the non-Ableian nature of MZMs.