Nonlinear scaling variable at the lower critical dimension: Scaling in the 2D random field Ising model\(^1\) LORIEN HAYDEN, JAMES SETHNA, Cornell Univ — We systematically analyze the nonlinear invariant scaling variables at bifurcations in the renormalization-group flow, and apply our methods to the two-dimensional random-field Ising model (RFIM). At critical points, the universal scaling functions are usually written in terms of homogeneous invariant combinations of variables, like \(L t^\nu\) in the finite-size scaling form for the magnetization \(M(T|L) \sim t^{-\beta} M(Lt^\nu)\), where \(t \propto T_c - T\). The renormalization-group flow for the RFIM has a pitchfork bifurcation in two dimensions, where the correlation length has been argued to diverge exponentially, \(\xi \propto \exp^{1/2\Delta}\), leading to the invariant scaling combination \(L/\xi \sim L/\exp^{1/2\Delta}\). Our analysis, inspired by normal-form theory, suggests that this exponential divergence can take a richer, more general scaling form at a generic pitchfork bifurcation. We explore possible consequences for simulations.

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