Equilibrium theory for braided elastic filaments
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Motivated by supercoiling of DNA and other filamentous structures, we formulate a theory for equilibria of 2-braids, i.e.,
structures formed by two elastic rods winding around each other in continuous contact and subject to a local interstrand
interaction. Unlike in previous work no assumption is made on the shape of the contact curve. Rather, this shape is found
as part of the solution. The theory is developed in terms of a moving frame of directors attached to one of the strands with
one of the directors pointing to the position of the other strand. The constant-distance constraint is automatically satisfied
by the introduction of what we call braid strains. The price we pay is that the potential energy involves arclength derivatives
of these strains, thus giving rise to a second-order variational problem. The Euler-Lagrange equations for this problem give
balance equations for the overall braid force and moment referred to the moving frame as well as differential equations that
can be interpreted as effective constitutive relations encoding the effect that the second strand has on the first as the braid
deforms under the action of end loads. Simple analytical cases are discussed first and used as starting solutions in parameter
continuation studies to compute classes of both open and closed (linked or knotted) braid solutions.