Kahler Structures on Spaces of Framed Curves

TOM NEEDHAM, The Ohio State University — Moduli spaces of loops in Euclidean space have been studied from a variety of perspectives due to their applications to fluid dynamics, statistical physics of polymers and shape recognition. We consider the moduli space of Euclidean similarity classes of parameterized framed loops. This space is an infinite-dimensional Kahler manifold; in fact we show that it is isomorphic to an infinite-dimensional Grassmannian with a natural Kahler structure. This result gives connections between previous results on loop spaces by Younes-Michor-Shah-Mumford (in the context of image recognition) and Millson-Zombro (in the context of symplectic geometry). The moduli space has many interesting Hamiltonian group actions, such as the diffeomorphism group of the circle acting by reparameterizations. The geodesic distance between the orbits of this group can be computed very efficiently, giving a new algorithm for shape recognition of ring polymers and oriented trajectories.