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Pure and Poetic: Butterfly in the Quantum World

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Story of the Hofstadter butterfly is a magical occurrence in a quantum flatland of two-dimensional crystals in a magnetic field. In this drama, the magnetic flux plays the role of Planck constant, linking the variables x and p in the butterfly Hamiltonian $H = cosx + cosp$ as $[x, p] = i\hbar$. It is a story of reunion of Descartes and Pythagoras and tale of this quantum fractal is related to Integral Apollonian gaskets. Integers rule the butterfly landscape as quantum numbers of Hall conductivity while irrational numbers emerge as the asymptotic magnification of these topological integers in the kaleidoscopic images of the butterfly [1].

Simple variations of the above Hamiltonian generates a wide spectrum of physical phenomenon. For example, the Hamiltonian $H = \cos x + \lambda \cos p$ with the parameter $\lambda \neq 1$ in its zero energy solution *hides* the critical point of a topological transition in a superconducting chain and thus barely misses the Majorana fermions[2]. Another example is the Hamiltonian obtained by including terms like $\cos(x \pm p)$ which for flux *half* exhibits Dirac semi-metallic states in addition to all integer quantum Hall states corresponding to all possible solutions of the Diophantine equation for this value of the magnetic flux. In this analytically tractable model where the parameter λ varies periodically with time, the topological states are described by edge modes whose dispersion is given by a pure cosine function[3].

Finally, nature has composed beautiful variations of the Hofstadter butterfly not only in systems such as Penrose and Kagame lattices and also in the relativistic colorful world of quarks and antiquarks[4].

(1) "Butterfly in the Quantum World", Indubala I Satija with contributions by Douglas Hofstadter, IOP Concise, Morgan and Claypool publication, 2016.

(2) I. Satija and G. Naumis, Phys Rev B **88**, 054204 (2013)

(3) I. Satija and E. Zhao, arXiv:1609.02807

(4) G. Endrödi, 32nd Int. Symp. on Lattice Field Theory 2014, New York: Columbia University