

MAR17-2016-020305

Abstract for an Invited Paper
for the MAR17 Meeting of
the American Physical Society

Extending Landauers bound from bit erasure to arbitrary computation¹

DAVID WOLPERT, Santa Fe Institute

The minimal thermodynamic work required to erase a bit, known as Landauers bound, has been extensively investigated both theoretically and experimentally. However, when viewed as a computation that maps inputs to outputs, bit erasure has a very special property: the output does not depend on the input. Existing analyses of thermodynamics of bit erasure implicitly exploit this property, and thus cannot be directly extended to analyze the computation of arbitrary input-output maps. Here we show how to extend these earlier analyses of bit erasure to analyze the thermodynamics of arbitrary computations. Doing this establishes a formal connection between the thermodynamics of computers and much of theoretical computer science. We use this extension to analyze the thermodynamics of the canonical “general purpose computer considered in computer science theory: a universal Turing machine (UTM). We consider a UTM which maps input programs to output strings, where inputs are drawn from an ensemble of random binary sequences, and prove: i) The minimal work needed by a UTM to run some particular input program X and produce output Y is the Kolmogorov complexity of Y minus the log of the “algorithmic probability of Y . This minimal amount of thermodynamic work has a finite upper bound, which is independent of the output Y , depending only on the details of the UTM. ii) The expected work needed by a UTM to compute some given output Y is infinite. As a corollary, the overall expected work to run a UTM is infinite. iii) The expected work needed by an arbitrary Turing machine T (not necessarily universal) to compute some given output Y can either be infinite or finite, depending on Y and the details of T . To derive these results we must combine ideas from nonequilibrium statistical physics with fundamental results from computer science, such as Levins coding theorem and other theorems about universal computation.

¹I would like to acknowledge the Santa Fe Institute, Grant No. TWCF0079/AB47 from the Templeton World Charity Foundation, Grant No. FQXi-RH13-1349 from the FQXi foundation, and Grant No. CHE-1648973 from the U.S. National Science Foundation