Einstein and Quaternions in 1906: An Alternate History

DOUGLAS SWEETSER, None — 1905 was Einstein’s Annus mirabilis. Special relativity applies to observers traveling at a steady speed. For observers who accelerate, the difficult math called Riemann differential geometry is needed that took Einstein 10 years to master. What if instead, Einstein had been given the math tool known as quaternions (a way to add and multiply events)? Einstein’s great skill was in finding things that do not change in Nature. Square the difference in time and space measured between two events:

\[
(dt, dR_i/c)^2 = \left(dt^2 - dR_i^2/c^2, 2dtdR_i/c \right).
\]

For special relativity, the first term is invariant and the other three change. Imagine the three space-times-time terms are invariant but the interval changes. Here is one possibility:

\[
(dt, dR_i/c) \rightarrow (dt', dR'_i/c) = \left(e^{-GM/c^2R}dt, e^{GM/c^2R}dR_i/c \right)
\]

\[
(dt', dR'_i/c)^2 = \left(e^{-2GM/c^2R}dt^2 - e^{2GM/c^2R}dR'_i^2/c^2, 2dtdR_i/c \right).
\]

The Rosen metric is the first term, which is consistent with tests of weak gravitational fields. At 2nd order PPN accuracy, predictions will differ. A new invariant in Nature may provide an algebraic approach to gravity as useful as special relativity.

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None

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