Abstract Submitted
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Revisiting Lorentz Transformations with Quaternions DOUGLAS SWEETSER, Quaternions.com — Minkowski recognized that special relativity could be viewed as a rotation in a 4D vector space. Unit quaternions (the compact Lie group SU(2)) are a double cover for 3D rotations, SO(3). It was long claimed that representing the non-compact Lorentz group SO(3, 1) with quaternions could not be done. In 2010 I found a way to generalize a rotation to do Lorentz boosts (Dr. Kharinov discovered independently):

$$B \to B' = hBh^* + \frac{1}{2}((hhB)^* - (h^*h^*B)^*)$$

If $h = (\cosh(x), I\sinh(x))$, this do a Lorentz boost. In 2013 I noticed that for a quaternion cross product normalized to one, the scalar term is zero and the second and third terms cancel leaving the 3D rotation. Physics cannot be done with space-time alone. Space-time is a base space and an affine space, energy-momentum. Three rotations live in space-time, three velocities in energy-momentum. View the quaternion scalar as time and the 3-vector as space, so animations of SU(2) and SO(3, 1) can be created. SU(2) starts as a point, specifically t=-1 at the spatial origin. It grows to its maximum size at time-now, t=0. The sphere shrinks to zero size at t=1. The animation for SO(3, 1) starts out infinitely huge, shrinks to its smallest size at t=0 matching SU(2) before expanding to infinity.

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