Approximate Solution of the Time-Independent Schrödinger Equation for the Quartic Oscillator

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Exact solutions to the quantum quartic oscillator are not known. We equip the set of Hamiltonian operators with a metric, thereby providing a notion of distance between these operators. This metric is a generalization of the $L^2$ metric on the space of Lebesgue measurable functions. We determine Hamiltonian operators with known solutions (to the Schrödinger equation), and then use the generalized metric to find a unique Hamiltonian with known solutions that is minimal in distance to the quartic Hamiltonian. The Hamiltonian that we seek is, in fact, the Hamiltonian of the harmonic oscillator. Minimizing the distance will correspondingly suggest a harmonic frequency. The approximate solutions to the quartic oscillator will thus be the solutions of the harmonic oscillator with the suggested frequency.