A Theoretical Estimate for the Frequency of the TNL Oscillator
\[ \ddot{x} + x + x^{1/3} = 0 \]

DORIAN WILKENSON, RONALD MICKENS, Clark Atlanta University — Truly nonlinear (TNL) oscillators have the property of having no linear approximation at the fixed-point of the modeling differential equation [1]. For a conservative oscillator this means that the fixed-point is a nonlinear center. Another feature of TNL oscillators is that none of the standard perturbation expansion procedures can be applied to calculate analytical approximations to the periodic or oscillator solutions [2]. Using the initial conditions \( x(0) = A \) and \( \dot{x}(0) = 0 \), we calculate the frequency \( \omega(A) \) of the equation given in the title for small, \( 0 < A \ll 1 \), and large \( A \gg 1 \), amplitudes. From these expressions a composite function, \( \omega(A) \), is found such that these two special limits hold. This function should provide an accurate estimate of the frequency for the full range of amplitude values, i.e., \( 0 < A < \infty \).


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