Generating Geometrical Elements for Any Space-Time

DENNIS MARKS, Valdosta State U. — To distinguish time from space, use real Clifford algebras $\mathbb{R}_{n:s}$, where $n$ is the number of dimensions and $s$ is their signature ($s = -n, -n+2, \ldots, or n$). $\mathbb{R}_{n:s}$ is isomorphic to algebras of real, complex, or quaternionic matrices $\mathbb{R}(2^{n/2})$, $\mathbb{C}(2^{n-1}/2)$, or $\mathbb{H}(2^{n-3}/2)$, or of block diagonal matrices $^2\mathbb{R}(2^{n-1}/2)$ or $^2\mathbb{H}(2^{n-3}/2)$, for $|(s + 3) \mod 8 - 4| = 1, 2, 3, 0, or 4$, respectively. Each of the $n$ basis vectors $e_\nu$ satisfies $e_\mu \cdot e_\nu = \eta_{\mu\nu}I_{n:s}$, where the $e_\nu$ are orthogonal $\eta_{\mu\nu} = 0$ for $\mu \neq \nu$ and normalized $\eta_{\mu\nu} = +1$ for $p$ space-like dimensions and $\eta_{\mu\nu} = -1$ for $q$ time-like dimensions) and where $I_{n:s}$ is the identity matrix whose rank is given by the isomorphisms above. The geometrical elements are the scalar $I_{n:s}$, basis vectors $e_\nu$, and their products (bivectors, trivectors, etc.) up to the pseudo-scalar $n$-volume $J_{n:s} = e_0 e_1 \cdots e_{n-1}$. Now $(J_{n:s})^2 = (-1)^{s(s-1)/2} I_{n:s} = \sigma_s I_{n:s}$. The direct product of $\mathbb{R}_{n:s}$, with $n$ orthonormal basis vectors $e_\nu$ with signature $s$, and $\mathbb{R}_{n':s'}$, with $n'$ orthonormal basis vectors $e_{\nu'}$ with signature $s'$, is $\mathbb{R}_{n+n':s+s'}$. Orthonormal basis vectors $e_\nu \otimes I_{n':s'}, J_{n:s} \otimes e_{\nu'}$ with signature $s + s'\sigma_s$, for even positive $n$. Orthonormal basis vectors for any positive $n$ with any possible signature can be generated from the two orthonormal basis vectors of the Minkowskian plane.

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