Exact Results on Potts Model in a Generalized External Field  

YAN XU, Department of Physics at Florida State University, ROBERT ELLSWORTH SHROCK, C. N. Yang Institute for Theoretical Physics at Stony Brook University — The $q$-state Potts model is a spin model that has been of longstanding interest as a many-body system in statistical physics. A natural generalization is to consider this model in a generalized external field that favors or disfavors spin values in a subset $I_s = \{1, \ldots, s\}$ of the total set of $q$-state spin values.

We obtain a powerful exact formula (Shrock formula) for the partition function of this generalized Potts model on various families of graphs $G$, $Z(G, q, s, v, w)$, where $v$ and $w$ are temperature- and field-dependent Boltzmann variables. An important property of this formula is that it expresses $Z(G, q, s, v, w)$ in a graph-theoretic manner as a sum of contributions from spanning subgraphs $G'$ of the graph $G$, rather than as a sum over spin configurations. Using this general formula, we derive a number of exact properties of $Z(G, q, s, v, w)$. We also analyze an interesting special case of the zero-temperature Potts antiferromagnet, corresponding to a set-weighted chromatic polynomial $Ph(G, q, s, w) \equiv Z(G, q, s, -1, w)$ that counts the number of colorings of the vertices of $G$ subject to the condition that colors of adjacent vertices are different, with a weighting $w$ that favors or disfavors colors in the interval $I_s$. 

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