

Abstract Submitted  
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**Derivation of Boltzmann's Principle** DONALD H. KOBE, University of North Texas, MICHELE CAMPISI<sup>1</sup>, University of Augsburg — Using a classical mechanical model of thermodynamics, we derive Boltzmann's Principle for the entropy  $S_B = k_B \ln W$ , where  $k_B$  is Boltzmann's constant and  $W$  is the number of microstates compatible with an energy  $E$ . The argument is based on the heat theorem which is the combined first and second laws of thermodynamics. It dates back to the work of Helmholtz and Boltzmann, but the argument has remained almost unknown. We first discuss a one-dimensional model. The phase-space volume entropy, microcanonical distribution, and ergodic theorem naturally emerge for one dimension. The argument is then generalized to an arbitrary number of  $N$  particles. Using the ergodic hypothesis, we show that the entropy is  $S = k_B \ln \Phi$ , where  $\Phi$  is the phase-space volume enclosed by a hypersurface of energy  $E$ . For very large systems with  $N \gg 1$ , the volume entropy  $S$  approaches the surface entropy  $S \approx s = k_B \ln(\Omega dE)$ , where  $\Omega = \partial\Phi/\partial E$  is the density of states on the hypersurface of energy  $E$  and  $dE$  is an irrelevant constant. For cells in phase space the size of Planck's constant  $h$  the density of states  $\Omega \approx W$ , which proves Boltzmann's Principle. However, the correct entropy for any number of particles is the phase-space volume entropy  $S$ .

<sup>1</sup>Based in part on my dissertation at the University of North Texas.

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