Noether’s Theorem and the Work-Energy Theorem for a Particle in an Electromagnetic Field

DONALD KOBE, University of North Texas — Noether’s theorem is based on two fundamental principles. The first is the extremum of the action and the second is the invariance of the action under infinitesimal continuous transformations. The first gives Hamilton’s principle of least action that results in the Euler-Lagrange equation. The second gives the Rund-Trautman identity for the generators of infinitesimal transformations. We apply these to a charged particle in an external electromagnetic field. The Euler-Lagrange equation gives the equation of motion. A solution of the Rund-Trautman identity for the generators is obtained by solving the generalized Killing equations. When the Euler-Lagrange equation and the Rund-Trautman identity are combined we obtain Noether’s theorem for a conserved quantity. Using the equation of motion and the generators of infinitesimal transformations for a charged particle in an external electromagnetic field, we obtain the work-energy theorem. Even though this theorem can be obtained directly from the equation of motion, this problem is a good example of using Noether’s theorem that is necessary for more complicated situations.

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