**Introduction to paths H and application of homotopy theory in physics**

FIDELE TWAGIRAYEZU, Texas State University — Firstly, we introduced the action of space operators on a regular interval to generate a variable interval. Secondly, we introduced the concept of a family T of paths H, and we showed that these paths are homotopic on a contractible space even though they do not have common endpoints. Finally, we applied the concept of paths H on a contractible space in physics.

Let $A$ be a subset of $X$. Let $I_{[a,b]}$ be a regular interval such that $I_{[a,b]} \subseteq A$, for $a, b \in A$. Let $(\alpha_{a,b})$ be space operators associated with $[a,b]$, then a variable interval is $I_{[x,y]} = (\alpha_{a,b})I_{[a,b]}$ such that $\{I_{[x,y]}\} \subseteq X$, $\min\{I_{[x,y]}\} = ax$, and $\max\{I_{[x,y]}\} = by$ for all $x, y \in X$. Let $X$ be a topological space. Let $f, g: [0,1] \to X$ be continuous paths for all $t \in [0,1]$. $T$ is the family of continuous paths $H: [0,1] \times [0,1] \to X$ such that $H(t,0) = f$, $H(t,1) = g$ for all $t \in [0,1]$, and $H(0,s_t) = f(0)$, $H(1,s_t) = f(1)$, $H(0,s_g) = g(0)$, $H(1,s_g) = g(1)$ for all $s_t, s_g \in [0,1]$. Such $f$ and $g$ are $H$-topic paths. If $X$ is contractible, then $H$ is a homotopy. In addition, if $s_t = s_g$, then $f(0) = g(0)$ and $f(1) = g(1)$, and the family $T$ of paths $H$ becomes the well-known homotopy of paths (with same endpoints). Let $M_G$ be a simply connected gravitational field. We showed that the Hamiltonian for free fall-paths on $M_G$ obeys the homotopy theory.

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