Lagrangian Densities and Principle of Least Action in Quantum Mechanics DONALD H. KOBE, Department of Physics, University of North Texas
— A Lagrangian density in terms of the wave function and its first derivatives can be used to derive the Schroedinger equation. From this Lagrangian density, we derive an equivalent one involving the Hamiltonian with second derivatives of the wave function that is commonly used in quantum mechanics. Using Hamilton’s Principle of Least Action, we obtain the time-dependent Schroedinger equation by varying with respect to the wave function. For a time-independent Hamiltonian and a stationary-state trial wave function, the Principle of Least Action gives the usual Rayleigh-Ritz energy variational principle. Using a Hartree product trial wave function for a many-boson system, we apply the Principle of Least Action to obtain the time-dependent Gross-Pitaevski equation, a nonlinear Schroedinger equation that describes a Bose condensate.