Analysis of Lagrange’s original derivation of the Euler-Lagrange Differential Equation

RYAN LAUGHLIN, HUNTER CLOSE, Texas State University - San Marcos — The Euler-Lagrange differential equation provides the Lagrangian equations of motion, and thus allows the exact trajectory of an object in a potential to be found. We analyze the original derivation of the Euler-Lagrange differential equation via a translation of the third edition of Lagrange’s Mecanique Analytique (1811). We compare and contrast this derivation with the derivation commonly done in a junior-level classical mechanics course. Lagrange uses several founding concepts to produce a generalized equation of motion for all dynamics. These concepts are, in the order addressed by Lagrange, the Principle of Virtual Velocities, the Conservation des Forces Vives, and the Principle of Least Action. Lagrange then employs what he calls the Method of Variations to the general equation of motion for dynamics to ultimately resolve something similar to the Euler-Lagrange Differential equation we use today. We also compare modern notation with Lagrange’s notation.

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