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The periodic table of real geometric algebras, bits of space-time, and the Standard Model. DENNIS MARKS, Valdosta State University — Real geometric algebras $\mathbf{R}_{n;s}$ in n dimensions with signature s are isomorphic to algebras of real, complex, or quaternionic matrices $\mathbf{R}(2^{\frac{\mathbf{n}}{\mathbf{2}}})$, $\mathbf{C}(2^{\frac{n-1}{2}})$, or $\mathbf{H}(2^{\frac{n-2}{2}})$, or of block diagonal matrices ${}^2\mathbf{R}(2^{\frac{n-1}{2}})$ or ${}^2\mathbf{H}(2^{\frac{n-3}{2}})$, for $|(s+3)_{\bmod 8} - 4| = 1$, 2, 3, 0, or 4, respectively. Only for *n = 2 or 4* and *s = 0 or 2* is $\mathbf{R}_{n;s}$ isomorphic to real $n \times n$ matrices $\mathbf{R}$(n). $\mathbf{R}_{2;2}$ and $\mathbf{R}_{2;0}$ describe the Euclidean plane and the Minkowskian plane. Their direct product, $\mathbf{R}_{4;2} = \mathbf{R}_{2;0} \otimes \mathbf{R}_{2;2}$, describes 4-d space-time with signature + + + − and with dynamical elements (position, spin, momentum, and action) that satisfy the Heisenberg commutation relations. Quantum mechanics emerges naturally. Electromagnetism, described by $U(1) \approx \mathbf{C} \approx \mathbf{R}_{1;-1}$, has one time-like coordinate; the weak force, described by *SU(2)* $\approx$ *SO(3)* $\approx \mathbf{R}_{3;3}$, has three space-like coordinates. Thus the real algebra of the symmetry group of the electro-weak force is isomorphic to the real algebra of space-time. Finally, $\mathbf{R}_{8;2} = \mathbf{R}_{4;0} \otimes \mathbf{R}_{4;2}$ is isomorphic to $\mathbf{R}$(16), into which can be fit three generations of weakly interacting Fermi doublets and three generations of three colors of quarks. Every 8 dimensions thereafter, geometric algebras factor into direct products of $\mathbf{R}$(16), interpreted as a 4-d hexadecimal space-time lattice with four additional internal coordinates for the Standard Model.

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