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The Connection between Noneuclidean Geometry and Special Relativity in an Expanding Universe FELIX T. SMITH, SRI International — The homogeneous Lorentz group is also the isometry group of noneuclidean geometry in hyperbolic space, but the connection has not been fully exploited in special relativity. In a 1907 lecture Minkowski recognized that the velocity $\mathbf{v}$ in special relativity generates a noneuclidean manifold. He soon showed this to be part of a covariant 4-vector $\mathsf{w} = \left(1 - v^2/c^2\right)^{-1/2} (v_x, v_y, v_z, ic)$, the vector to the 3-surface of a 4-sphere of imaginary radius ic in velocity space. Unable to identify a comparable geometry in position space, he omitted all mention of this velocity symmetry in later publications. Had the Hubble expansion (1927) been known, he could have used the Hubble time t_H, a cosmic time variable $t = t_H + \delta t$, and a position 4-vector $\mathsf{s} = (t/t_H) \left(x, y, z, i\left[c^2 {t_H}^2 + r^2\right]^{1/2}\right)$, an expanding hypersphere of imaginary radius $R(t) = ict$. The interval between two local events is, to first order, $\Delta r = (\Delta x, \Delta y, \Delta z, ic\Delta t)$. This is the Minkowski 4-vector in differential form, but the source of its imaginary time is identified as the cosmological expansion. An extended Lorentz group follows if the 4-vectors are replaced by tensors of position and velocity.

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