Abstract Submitted
for the DFD08 Meeting of
The American Physical Society

Determining the stability of steady inviscid flows through "Imperfect Velocity-Impulse" diagrams P. LUZZATTO-FEGIZ, C.H.K. WILLIAMSON, Cornell University — More than a century ago, Lord Kelvin proposed a variational argument for determining the stability of steady inviscid flows; while the underpinnings of the method are well established, its application has been the subject of extensive debate. Considering, for example, a vortex configuration rotating at a rate Ω with impulse J and energy E, Kelvin argued that an equilibrium corresponds to a stationary point of $H = E - \Omega J$. Since H is conserved, the second variation $\delta^2 H$ constrains the dynamics and can be used to assess stability. Unfortunately, computation of $\delta^2 H$ is often impossible or impractical. Saffman & Szeto (1980) suggested that extrema in a plot of E vs J could be used to identify changes in $\delta^2 H$. However, Dritschel (1985) later pointed out the lack of a firm link between $\delta^2 H$ and a plot of E vs J. Furthermore, he stated that even if such link could be proven, changes of stability could also occur, at bifurcations, away from extrema in E and J. We address both issues by proposing a new approach. We introduce a theorem from dynamical systems theory to prove that extrema in a plot of J vs Ω (instead of E vs J) are indeed related to the properties of $\delta^2 H$, while we use ideas from imperfection theory to ensure that bifurcations are detected by means of an "imperfect velocity-impulse" (IVI) diagram. By applying our approach to several classical flows, we obtain stability results in agreement with linear analysis, while additionally discovering new steady solutions.

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Date submitted: 01 Aug 2008 Electronic form version 1.4