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Andreas Acrivos Dissertation Prize Lecture: Stability of inviscid flows from bifurcation diagrams exploiting a variational argument[1]

PAOLO LUZZATTO-FEGIZ, Woods Hole Oceanographic Institution

Steady fluid solutions play a special role in the dynamics of a flow: stable states may be realized in practice, while unstable ones may act as attractors. Unfortunately, determining stability is often a process far more laborious than finding steady states; indeed, even for simple vortex or wave flows, stability properties have often been the subject of debate. We consider here a stability idea originating with Lord Kelvin (1876), which involves using the second variation of the energy, $\delta^2 E$, to establish bounds on a perturbation. However, for numerically obtained flows, computing $\delta^2 E$ explicitly is often not feasible. To circumvent this issue, Saffman & Szeto (1980) proposed an argument linking changes in $\delta^2 E$ to turning points in a bifurcation diagram, for families of steady flows. Later work has shown that this argument is unreliable; the two key issues are associated with the absence of a formal turning-point theory, and with the inability to detect bifurcations (Dritschel 1995, and references therein). In this work, we build on ideas from bifurcation theory, and link turning points in a velocity-impulse diagram to changes in $\delta^2 E$; in addition, this diagram delivers the direction of the change of $\delta^2 E$, thereby providing information as to whether stability is gained or lost. To detect hidden solution branches, we introduce to these fluid problems concepts from imperfection theory. The resulting approach, involving "imperfect velocity-impulse" diagrams, leads us to new and surprising results for a wide range of fundamental vortex and wave flows; we mention here the calculation of the first steady vortices without any symmetry, and the uncovering of the complete solution structure for vortex pairs. In addition, we find precise agreement with available results from linear stability analysis.

[1]Doctoral work advised by C.H.K. Williamson at Cornell University.