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Generalized Courant-Snyder theory and Kapchinskij-Vladimirskij distribution for high intensity beams in coupled transverse focusing lattices[1]

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Courant-Snyder (CS) theory gives a complete description of the uncoupled transverse dynamics of charged particles in electromagnetic focusing lattices. In this paper, CS theory is generalized to the case of coupled transverse dynamics with two degree of freedom. The generalized theory has the same structure as the original CS theory for one degree of freedom. The four basic components of the original CS theory, i.e., the envelope equation, phase advance, transfer matrix, and the CS invariant, all have their counterparts in the generalized theory. The envelope function is generalized into an envelope matrix, and the envelope equation becomes a matrix envelope equation with matrix operations that are noncommutative. The generalized theory gives a new parameterization of the 4D symplectic transfer matrix that has the same structure as the parameterization of the 2D symplectic transfer matrix in the original CS theory. This parameterization can provide a valuable framework for accelerator design and particle simulation studies. For example, it is discovered that the stability of coupled dynamics is completely determined by the generalized phase advance. Two stability criteria are given, which recover the known results about sum and difference resonances in the weakly coupled limit. In an uncoupled lattice, the Kapchinskij-Vladimirskij distribution function first analyzed in 1959 is the only known exact solution of the nonlinear Vlasov-Maxwell equations for high-intensity beams including selffields in a self-consistent manner. The Kapchinskij-Vladimirskij solution is generalized to high-intensity beams in a coupled transverse lattice using the generalized Courant-Snyder invariant for coupled transverse dynamics. This solution projects to a rotating, pulsating elliptical beam in transverse configuration space, determined by the generalized matrix envelope equation.

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