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Probability Current in Hydrogen with Spin-Orbit Interaction

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$$\mathcal{H} = \frac{\mathbf{p^2}}{\in\Updownarrow} + \mathcal{V}(\mathbf{x}) + \frac{\nabla \mathbf{V}(\mathbf{x}) \times \mathbf{p}}{\mathbf{2m^2c^2}} \cdot \mathbf{S}. \tag{1}$$

To describe the flow of probability in this system, we derive the continuity equation, which takes the usual form. In this case, however, we find the probability current density $\mathbf{j}(\mathbf{x}, \mathbf{t})$ to be the sum of two terms. The first term is the one obtained by most quantum mechanics textbooks during their derivation of the continuity equation. The second term,

$$\mathbf{j_s}(\mathbf{x}, \mathbf{t}) = \frac{\mathbf{1}}{\mathbf{2m^2c^2}} \sum_{\sigma, \sigma' = \uparrow, \downarrow} \Big[\psi^*(\mathbf{x}, \sigma, \mathbf{t}) \langle \sigma | \mathbf{S} | \sigma' \rangle \psi(\mathbf{x}, \sigma', \mathbf{t}) \Big] \times \nabla \mathbf{V}(\mathbf{x}), \tag{2}$$

arises due to the inclusion of the spin-orbit term in the Hamiltonian and is small compared to the first. Using a perturbative treatment, we calculate $\mathbf{j}(\mathbf{x}, \mathbf{t})$ for hydrogenlike atoms; for states with $\ell = 0$, we find that $\mathbf{j}(\mathbf{x}, \mathbf{t}) = \mathbf{j_s}(\mathbf{x}, \mathbf{t})$.

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