Abstract Submitted
for the MAR13 Meeting of
The American Physical Society

Overlooked restrictions on Euler angles in quantum computation[1]

MITSURU HAMADA, Tamagawa University — Let X, Y, Z denote the Pauli matrices. For $\vec{n} = (n_x, n_y, n_z) \in \mathbf{R}^3$ with $n_x^2 + n_y^2 + n_z^2 = 1$ and $\theta \in \mathbf{R}$, put $R_{\vec{n}}(\theta) = \cos(\theta/2)I - i\sin(\theta/2)(n_x X + n_y Y + n_z Z)$. Put $R_y(\theta) = R_{(0,1,0)}(\theta)$ and $R_z(\theta) = R_{(0,0,1)}(\theta)$. Theorem: Assume $\alpha, \gamma, \theta \in \mathbf{R}$, $\vec{n} = (n_x, n_y, n_z) \in \mathbf{R}^3$ and $n_x^2 + n_y^2 + n_z^2 = 1$. Then, there exists some $\beta, \delta \in \mathbf{R}$ satisfying $R_{\vec{n}}(\theta) = e^{i\alpha} R_z(\beta) R_y(\gamma) R_z(\delta)$ if and only if (iff) $e^{i\alpha} = 1$ or -1, and $\sqrt{1 - n_z^2}|\sin(\theta/2)| = |\sin(\gamma/2)|$. Corollary: Assume $\alpha, \gamma \in \mathbf{R}$, $\vec{n} = (n_x, n_y, n_z) \in \mathbf{R}^3$ and $n_x^2 + n_y^2 + n_z^2 = 1$. Then, there exist some $\beta, \delta, \theta \in \mathbf{R}$ such that $e^{i\alpha} R_z(\beta) R_{\vec{n}}(\theta) R_z(\delta) = R_y(\gamma)$ iff $e^{i\alpha} = 1$ or -1, and $|\cos(\gamma/2)| \geq |n_z|$. This corollary shows a widespread fallacy on universal gates in quantum computation. Namely, when $|\cos(\gamma/2)| < |n_z| < 1$, according to a claim often found in textbooks, $R_y(\gamma)$ could be written as $e^{i\alpha} R_z(\beta) R_{\vec{n}}(\theta) R_z(\delta)$ for some $\alpha, \beta, \delta, \theta \in \mathbf{R}$. This is untrue by the corollary.

[1] Supported by SCOPE, KAKENHI 22540150 and 21244007

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Date submitted: 21 Dec 2012 Electronic form version 1.4