Abstract Submitted
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Geometry in a dynamical system without space: Hyperbolic Geometry in Kuramoto Oscillator Systems[1] JAN ENGELBRECHT, BOLUN CHEN, RENATO MIROLLO, Boston College — Kuramoto oscillator networks have the special property that their time evolution is constrained to lie on 3D orbits of the Möbius group acting on the N-fold torus T^N which explains the $N-3$ constants of motion discovered by Watanabe and Strogatz. The dynamics for phase models can be further reduced to 2D invariant sets in T^{N-1} which have a natural geometry equivalent to the unit disk Δ with hyperbolic metric. We show that the classic Kuramoto model with order parameter Z_1 (the first moment of the oscillator configuration) is a gradient flow in this metric with a unique fixed point on each generic 2D invariant set, corresponding to the hyperbolic barycenter of an oscillator configuration. This gradient property makes the dynamics especially easy to analyze. We exhibit several new families of Kuramoto oscillator models which reduce to gradient flows in this metric; some of these have a richer fixed point structure including non-hyperbolic fixed points associated with fixed point bifurcations.

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