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Patterns through elastic instabilities, from thin sheets to twisted ribbons

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Sheets embedded in a given shape by external forces store the exerted work in elastic deformations. For pure tensile forces, the work is stored as stretching energy. When the forces are compressive, several ways to store the exerted work, combining stretching and bending deformations can be explored. For large deflections, the ratio of bending, $\sim Eh^3\zeta^2/L^4$ and stretching, $\sim Eh\zeta^4/L^4$ energies, suggests that strain-free solutions should be favored for thin sheets, provided $\zeta^2 \gg h^2$ (where $E, \zeta, Landh$ are the elastic modulus, the deflection, a characteristic sheet size and its thickness). For uniaxially constrained sheets deriving from the Elastica, strain-free solutions are obvious, i.e., buckles, folds or wrinkles grow to absorb the stress of compression. In contrast, crumpled sheets exhibit "origami-like" solutions usually described as an assembly of flat polygonal facets delimitated by ridges focusing strains are observed. This type of solutions is particularly interesting since a faceted morphology is isometric to the undeformed sheet, except at those narrow ridges. In some cases however, the geometric constraints imposed by the external forces do not allow solutions with negligible strain in the deformed state.

For instance, considering a circular sheet on a small drop, so thin that bending becomes negligible, i.e., $Eh^3/\gamma L^2 \ll 1$. The capillary tension, γ at the edge forces the sheet to follow the spherical shape of the drop. Depending on the magnitude of the capillary tension with respect to the stretching modulus, such a sheet on a sphere can be in full tension or subjected to azimuthal compression. These spherical solutions could generate a hoop stress of compression within a small strip at the sheet's edge. The mechanical response of the sheet will generate tiny wrinkles decorating the edge to relax the compression stress while keeping its spherical shape. Finally, twisting a paper ribbon under high tension spontaneously produces helicoidal shapes that also reflect stretching and bending deformations. When the tension is progressively relieved, longitudinal and transverse compressive stresses build. To relax the longitudinal stress while keeping the helicoid shape, the ribbons produce wrinkles that ultimately becomes sharp folds similar to the ridge singularities observed in crumpled paper. The relaxation of the transverse compression stress produces cylindrical solutions. All these examples illustrates the natural tendency of an elastic sheet to stay as close as possible to the imposed shape, i.e. flat, spherical, helicoid. The mechanical response of the elastic sheet aims to relieve the compressive stress by growing a given micro-structure, i.e. wrinkles, singularities. In this talk, we will explore the general mechanisms at work, based on geometry and a competition between various energy terms, involving stretching and bending modes.