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Bumping into the Butterfly, When I Was But a Bud

DOUGLAS HOFSTADTER, Center for Research on Concepts and Cognition, Indiana University, Bloomington

I will recount the main events that led me to discover the so-called "Hofstadter butterfly" when I was a physics student, over 40 years ago. A key moment in the tale was when, after years of futile struggle, I finally abandoned particle physics, and chose, with much trepidation, to try solid-state physics instead, a field of which I knew nothing at all. I was instinctively drawn to a long-standing classic unsolved problem in the field — What is the nature of the energy spectrum of Bloch electrons in a magnetic field? — when Professor Gregory Wannier told me that it involved a weird distinction between "rational" and "irrational" magnetic fields, which neither he nor anyone else understood. This mystery allured me, as I was sure that the rational/irrational distinction cannot possibly play a role in physical phenomena. I tried manipulating equations for a long time but was unable to make any headway, and so, as a last resort, I wound up using brute-force calculation instead. I programmed a small desktop computer to give me numbers that I then plotted by hand on paper, and one fine day, to my shock, my eyes suddenly recognized a remarkable type of pattern that I had discovered twelve years earlier, when I was an undergraduate math major exploring number theory. All at once, I realized that the theoretical energy spectrum I'd plotted by hand consisted of infinitely many copies of itself, nested infinitely deeply, and it looked a little like a butterfly, whence its name. This unanticipated discovery eventually led to many new insights into the behavior of quantum systems featuring two competing periodicities. I will briefly describe some of the consequences I found back then of the infinitely nested spectrum, and in particular how the baffling rational/irrational distinction melted away, once the butterfly's nature had been deeply understood.