Grad-Shafranov equation for non-axisymmetric MHD equilibria

JOSHUA BURBY, Los Alamos National Laboratory, NIKOS KALLINIKOS, ROBERT MACKAY, University of Warwick, SIMONS COLLABORATION ON HIDDEN SYMMETRIES AND FUSION ENERGY COLLABORATION — The structure of static MHD equilibria that admit continuous families of Euclidean symmetries is well understood. Such field configurations are governed by the classical Grad-Shafranov equation, which is a single elliptic PDE in two space dimensions. By revealing a hidden symmetry, we show that in fact all nondegenerate solutions of the equilibrium equations satisfy a generalization of the Grad-Shafranov equation. In contrast to solutions of the classical Grad-Shafranov equation, solutions of the generalized equation are not automatically equilibria, but instead only satisfy force balance averaged over the one-parameter hidden symmetry. We then explain how the generalized Grad-Shafranov equation can be used to reformulate the problem of finding exact three-dimensional smooth solutions of the equilibrium equations as finding an optimal volume-preserving symmetry.

1Research presented in this article was supported by the Los Alamos National Laboratory LDRD program under project number 20180756PRD4 and by the Simons Foundation (601970, RSM).

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Date submitted: 29 Jun 2020 Electronic form version 1.4