Inference of Schrödinger’s Equation from Classical Wave Mechanics[1] P-I. JOHANSSON, Uppsala University, SWE, J.X. ZHENG-JOHANSSON, IOFPR, SWE — A localized oscillatory point charge \( q \) generates in a one-dimensional box electromagnetic waves which may be generally described by monochromatic plane waves \( \{ \varphi_i = C_K e^{i(KX - \Omega T + \alpha_i)} \} \) of angular frequency \( \Omega \), wavevector \( K = \Omega/c \), velocity (of light) \( c \), and initial phases \( \{ \alpha_i \} \). \( q \) and \( \{ \varphi_i \} \) as a whole is here taken as a particle, which total energy \( E \) and mass \( M \) are given by the basic equations
\[
E = \hbar \Omega = Mc^2, \quad 2\pi \hbar
\]
being Planck constant. (For example, \( q = -e \) and \( M = 511 \) keV give an electron.) \( \{ \varphi_i \} \) as incident and reflected and those from the charge as reflected in the box superimpose into a total wave \( \psi = \sum \varphi_i \) that, as with \( \varphi_i \), obeys the classical wave equation (CWE):
\[
c^2 \frac{d^2 \psi}{dX^2} = \frac{d^2 \psi}{dT^2}
\]
If now the particle is traveling at velocity \( v \), in a potential field \( V = 0 \) here (see Ref. 2004b for \( V \neq 0 \)), then \( \{ \varphi_i' \} \) are Doppler effected and form a total wave \( \psi' = \Phi \Psi \), with \( \Psi = C \sin(K_dX)e^{i\Omega_dT} \) being the envelope about a beat wave and identifiable as de Broglie wave of angular frequency \( \Omega_d = \Omega(v/c)^2 \), and \( \Phi \) an undisplaced monochromatic wave. Using \( \psi' \) in CWE gives upon decomposition a separate equation describing the particle dynamics,
\[
-\frac{\hbar^2}{2M} \frac{\partial^2 \varphi(X,T)}{\partial X^2} = i\hbar \frac{\partial \Phi(X,T)}{\partial T},
\]
which is equivalent to Schrödinger’s equation.


Dr. J.X. Zheng-Johansson
IOFPR, Nykoping

Date submitted: 30 Nov 2004
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