Phase transition between quantum spin Hall and ordinary insulating phases SHUICHI MURAKAMI, Tokyo Institute of Technology and PRESTO, JST, SATOSHI ISO, KEK, YISHAI AVISHAI, Ben-Gurion University and RTRA project, LPS and CEA, MASARU ONODA, CERC, AIST and CREST, JST, NAOTO NAGAOSA, University of Tokyo, CREST, JST, and CERC, AIST — We theoretically study the phase transition between the quantum spin Hall (QSH) and insulator phases, which involves a change of the $Z_2$ topological number. We deal with 2D and 3D systems without impurity and interaction. We introduce a parameter $m$ controlling the phase transition, and we study whether the gap closes or not by one-parameter tuning. In general, level repulsion prevents the gap from closing. In fact, the physics of the $Z_2$ topological number is encoded in the problem whether the gap closes by tuning a single parameter. In 2D [1], as well as in the 3D inversion-symmetric systems [2], the gap closes at one point, $m = m_0$, whereas in 3D inversion-asymmetric systems [2], there appears a finite regime for $m$ ($m_1 < m < m_2$) showing a gapless phase between the QSH and insulator phases. In the gapless phase the monopoles and antimonopoles exist in the $k$-space, and they annihilate in pair at the gap opening. Implications of the present results to real materials [3] are discussed. [1] S. Murakami et al., Phys. Rev. B76, 205304 (2007). [2] S. Murakami, New J. Phys. 9, 356 (2007). [3] S. Murakami, Phys. Rev. Lett. 97, 236805 (2006).